Theorem 1. *Suppose* $a_0, a_1, \ldots, a_n$ *are real numbers and* $a_n \neq 0$. *then*

$$\sum_0^n a_j \frac{\partial^j X}{\partial^n x} = 0$$

is a homogeneous constant coefficient linear differential equation with characteristic polynomial

$$\sum_0^n a_j r^j \tag{1}$$

whose general solution is given by

$$X(x) = \sum_0^n C_j y_j(x)$$

where the C_j *are arbitrary constants and*

To each distinct root, r, *of* (1) *the* y_j *are the functions given by*

- *to each real simple root* r, *set the corresponding function* $y_j = e^{rx}$
- *to simple complex conjugate roots* $r = a \pm bi$ *set the pair of corresponding functions* $y_j = e^{ax}cos(bx)$ *and* $y_j = e^{ax}sin(bx)$
- *to each real root* r *of multiplicity* k *set the corresponding* k *functions* $y_j = x^i e^{rx}$ *for* $i = 0, 1, 2, \ldots k$
- *to each pair of complex conjugate roots* $r = a \pm bi$ *of multiplicity* k *set the corresponding* $2k$ *functions* $y_j = x^i e^{ax}cos(bx)$ *and* $y_j = x^i e^{ax}sin(bx)$ $i = 0, 1, 2, \ldots k$.

