1 Linear Kinematics

Suppose a rigid body moves along a line beginning at time 0, with constant acceleration a, initial position s_0, and initial velocity v_0. Then its position at time t, s_t, and its velocity at time t, v_t, satisfy the four key equations of translational kinematics.

1. $s_t - s_0 = \frac{1}{2}at^2 + v_0 t$
2. $v_t - v_0 = at$
3. $v_t^2 - v_0^2 = 2a(s_t - s_0)$
4. $s_t - s_0 = \left(\frac{v_0 + v_t}{2}\right) t$

It is convenient to express these equations in terms of the change in position, Δs, in velocity, Δv, and the square of velocity, Δv^2.

1'. $\Delta s = \frac{1}{2}at^2 + v_0 t$

2'. $\Delta v = at$

3'. $\Delta v^2 = 2a\Delta s$

4'. $\Delta s = \left(\frac{v_0 + v_t}{2}\right) t$

Often, the motion is motion of an object near the surface of the earth falling under the influence of the gravitational attraction of the earth. In this case, the constant acceleration is $-g = -9.8m/\sec^2$. The most common problems you will encounter require you to think about the physical situation.

For example, if the problem asks for the maximum height of a ball thrown in the air you have to realize that at the time it reaches its highest point its velocity is 0. Notice how easy it is to use [2] to calculate that the time is $\frac{v_0}{g}$ and then use [1] to show that its height at that time is $s_0 + \frac{3v_0^2}{2g}$.

Similarly, if a ball is thrown straight upward from the ground, it lands on the ground again at the time t when its displacement, $s_t - s_0$, is zero. Notice how easy it is to use [4] to show that the velocity at that instant is equal and opposite to its initial velocity and then to use [2] to show that the time it reaches the ground is $\frac{2v_0}{g}$.

Another idea that often plays a role in these problems is that the actual motion is in a plane rather than along a line. In that case one usually calculates the components of a and v_0 in the direction of convenient axes and applies [1] – [4] separately in the coordinate directions to obtain a final answer. Although this complicates the calculations the basic idea remains unchanged.

Questions

1. At what angle from horizontal should a ball be kicked to maximize the horizontal distance it travels? Assume the ball begins and ends on the ground.

2. A ball is kicked at a 50^o angle from horizontal. At what other angle from horizontal should the ball be kicked to land at the exact same spot? Assume the initial velocities are the same and both balls begin and end on the ground.

2 Angular Kinematics

Suppose a rigid body rotates about a fixed axis beginning at time 0, with constant acceleration α, initial angle θ_0, and initial angular velocity ω_0. Then its angular position at time t, θ_t, and its angular velocity at time t, ω_t, satisfy the four key equations of rotational kinematics.

1. $\theta_t = \frac{1}{2}\alpha t^2 + \omega_0 t + \theta_0$
2. $\omega_t = \alpha t + \omega_0$
3. $\omega_t^2 = \omega_0^2 + 2\alpha(\theta_t - \theta_0)$
4. $\theta_t - \theta_0 = \left(\frac{\omega_0 + \omega_t}{2}\right) t$

